THE FIRST AWARD OF THE LOBACHÉVSKI PRIZE.

THE Lobachévski prize is adjudged every three years. Its value is five hundred roubles. It is given for work in geometry, preferably non-Euclidean geometry. All works published within the six years preceding the award of the prize, and sent by their authors to the Physico-Mathematical Society of Kazan, are allowed to compete if published in Russian, French, German, English, Italian or Latin.

The Society has now in formal session awarded the prize to Sophus Lie, professor of mathematics at the University of Leipzig, for his work 'Theorie der Transformationsgruppen, Band III., Leipzig, 1893.' In this work the theory of non-Euclidean geometry has been exhaustively re-stated and re-established in a profound investigation of the work of Helmholtz on the space-problem.

To the genius of Helmholtz is due the conception of studying the essential characteristics of a space by a consideration of the movements possible therein.

But since the time when Helmholtz did his work on this subject the greatest of living mathematicians, Sophus Lie, formerly of Christiania, has enriched mathematics with a new instrument, the Theory of Groups, which its creator has applied with tremendous power to the Helmholtz treatment. Lie finds, as was almost inevitable, that certain details had escaped the great physicist, but that, with the tact of true genius, he had kept his main results free from error, though there comes to light a superfluity in his explicit assumptions, an unconscious assumption now seen to be mathematically important for the rigor of the demonstration, and at least one definite error in minor results.

Lie's method is in general the following. Consider a tri-dimensional space, in which a point is defined by three quantities, x, y, z.

A movement is defined by three equations:

$x' = f\ (x, y, z)$; $y' = \varphi\ (x, y, z,)$; $z' = \Psi\ (x, y, z)$.

By this transformation an assemblage, A, of points (x, y, z) becomes an assemblage, A′, of points (x', y', z').

This represents a movement which changes A to A′.

Now make, in regard to the space to be studied, the following assumptions:

1st. Assume: In reference to any pair of points which are moved, there is *something* which is left unchanged by the motion.

That is, after an assemblage of points, A, has been turned by a single motion into an assemblage of points, A′, there is a certain function, F, of the coordinates of any pair of the old points (x_1, y_1, z_1), (x_2, y_2, z_2) which equals that same function, F, of the corresponding new coordinates (x'_1, y'_1, z'_1), (x'_2, y'_2, z'_2); that is, $F\ (x_1, y_1, z_1, x_2, y_2, z_2) = F\ (x'_1, y'_1, z'_1, x'_2, y'_2, z'_2)$.

This *something* corresponds to the Cayley definition of the distance of two points when interpreted as completely independent of ordinary measurement by superposition of an unchanging sect as unit for length.

This independence, involving the determination of cross-ratio without any use of ordinary ratio, without using congruence, without using motion, Cayley never clearly saw. It follows from the profound pure projective geometry of von Staudt.

2d. Assume: If one point of an assemblage is fixed, every other point of this assemblage, *without any exception*, describes a surface (a two-dimensional aggregate).

When two points are fixed a point in general (exceptions being possible) describes a curve (a one-dimensional aggregate). Finally, if three arbitrary points are fixed, all are fixed (exceptions being possible). With these assumptions Lie proves exhaustively that the general results

of Helmholtz and Riemann follow; that is, there are three, and only three, spaces which fulfill these requirements, namely, the traditional, or Euclidean space, and the spaces in which the group of movements possible is the projective group transforming into itself one or the other of the surfaces of the second degree

$$x^2 + y^2 + z^2 \pm 1 = 0.$$

In the appreciation of this work of Lie's, prepared for the Society by Felix Klein, for which the Lobachévski gold medal was given him, he says that Lie's work stands out so prominently over all the others to be compared with it that a doubt as to the award of the prize would scarcely have been possible. Decisive for this judgment as to the height of the scientific achievement is not only the extraordinary depth and keenness with which Lie, in the fifth section of his book, handles what he has called the Riemann-Helmholtz space problem, but especially the circumstance that this treatment appears, so to say, as logical consequence of Lie's long-continued creative work in the province of geometry, especially his theory of continuous transformation groups.

The extraordinary importance which the works of Lie possess for the general development of geometry can scarcely be overestimated. In the coming years they will be still more widely prized than hitherto. Passing, then, to the consideration of the present state of the space question, Klein takes up the origin of axioms. Whence come the axioms? A mathematician who knows the non-Euclidean theories would scarcely maintain the position of earlier times that the axioms as to their concrete content are necessities of the inner intuition.

What to the uninitiated appears as such necessity shows itself, after long occupation with the non-Euclidian problems, as the result of very complex processes, and especially education and habit.

Do the axioms come from experience? Helmholtz energetically says yes! as is well known. But his expositions seem in a definite direction incomplete.

One will, in thinking over these, willingly admit that experience plays an important part in the formation of axioms, but will notice that just the point especially interesting to the mathematician remains untouched by Helmholtz.

It is a question of a process which we always complete in exactly the same way in the theoretical handling of any empirical data, and which, therefore, may seem quite clear to the scientist.

Expressed generally: *Always the results of any observations hold good only within definite limits of precision and under particular conditions; when we set up the axioms we put in the place of these results statements of absolute precision and generality.*

In this 'idealizing' of empirical data lies, in my opinion, the peculiar essence of axioms. Therein our addition is limited in its arbitrariness at first only by this, that it must cling to the results of experience and, on the other hand, introduce no logical contradiction.

Then enters as regulator also that which Mach calls the 'economy of thinking.' No one will rationally hold fast to a more complicated system of axioms when he sees that with a simpler system he already completely attains the exactitude requisite to the representation of the empirical data.

Klein goes on to mention the possibility of a series of topologically distinguishable space-forms built of limited (simply compendent) space-pieces either all Euclidean, all Lobachévskian or all Riemannian. Beside these three just mentioned family-types, the parabolic, the hyperbolic, the single elliptic, Klein has shown that the spherical, in which two geodetics always

cut in two points, is the only one which as a whole is freely movable in itself.

Then Klein says: "I consider all the topologically distinguished space-forms as equally compatible with experience. That in our theoretic considerations we prefer some of these space-forms (namely, the family types, that is, the properly parabolic, hyperbolic, elliptic) in order to finally assume the parabolic geometry, that is, the customary Euclidean geometry, as valid, happens simply from the principle of economy."

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*EARTHQUAKE SHOCKS IN GILES CO., VA.**

IMMEDIATELY following the earthquake of May 31, 1897, which was distinctly felt over most of the eastern portion of the United States, came newspaper reports of continued disturbance in the form of explosions and earth tremors in Giles county, Virginia. It was also reported that Mountain Lake had been drained, that the wells of Saltville, Virginia, had ceased to flow, and that large fissures had opened in the earth at various points in Giles county. At the urgent request of several citizens of Pearisburg, and with the idea that possibly there might be some foundation for the rumors afloat, I visited the region in the early part of June. The reports were found to be grossly exaggerated, as no disturbance had occurred at Mountain Lake, the Saltville wells were flowing as usual, and no fissures had appeared within the limits of Giles county. Under the circumstances the scientific results of my visit were insignificant, but there were certain phenomena observed which seemed to be worth recording.

The county of Giles lies on the northwestern side of the Appalachian Valley. Its surface is diversified by numerous ridges which cross the country from northeast to southwest. The rocks have been thrown into great folds, and are broken by numerous faults which also cross the region in the same direction. The principal object of my visit was to determine, if possible, whether there was any relation between the present disturbance and the geologic structure of the region; but, from the nature of the case, only a little information was obtained on the subject.

The earliest generally recognized earth tremor occurred on May 3. It loosened some bricks from old chimneys and was accompanied by considerable noise, like low rumbling thunder. From May 3 to 31 no shock of importance occurred, but many noises were heard, similar to the rumbling that accompanied the first quake. Many persons now believe that the same sort of noises occurred for a long time prior to May 3, but were passed unnoticed by the people, who, at that time, did not have their nerves wrought to such a tension that they heard and felt the slightest shock or earth tremor.

The shock of May 31 was probably more severe in and about Pearisburg than at any other point from which I have information. No serious damage was done even here, but old brick houses were badly shaken, and many chimneys were cracked and the topmost bricks hurled to the ground. Much noise accompanied this shock, and many of the inhabitants, already much disturbed by the previous heavy shock and the continued rumblings beneath them during the month, were terror-stricken. The noise did not stop with the main shock, but tremors and rumblings, or sharp reports, are described as occurring during the entire night following the shock. The intensity of these rumblings or reports varied according to location. Those of greatest severity were reported from the angle between Sugar Run and Pearis Mountains. Old veterans of the

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